Abdominal Aortic Aneurysm Wall Stress Analysis
from mathematical model to clinical application

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Introduction

Pathology
- Intervention and rupture risk

Mathematical model
- Constitutive models
- Linear vs non-linear material behavior
- Residual and initial stress
- Backward incremental method

Material properties
- Arterial wall
- Thrombus

Clinical study
- Medical imaging
- Compliance measurements
- Stress and strain analysis

Conclusions
properties AAA:

- local dilation of the aorta
- diameter over 3 cm
- mostly a-symptomatic
- rupture \(\Rightarrow\) mortality > 80%
- 13\textsuperscript{th} cause of death in western world
**Introduction**

Treatment:
- Open surgery
- Endovascular repair

Medical decision-making:
- Currently: diameter of AAA > 5 cm ⇒ intervention
- 30-day mortality rate of intervention ± 4%
- But: smaller aneurysms do rupture, larger aneurysms appear stable
- New possibility: decision also based on wall stress analysis
Introduction

patient specific modeling:

input needed:
- geometry of the AAA: MRI or CT
- aortic pressure wave
- constitutive model

hemodyn (PMS and TU/e)

Philips Medical Systems

TU/e
Mathematical model /constitutive models

fiber reinforced biological tissue: (Holtzapfel, van Ooijen, Driessen)

- neo-Hookean matrix (elastin)
- non-linear fibers (collagen)
- orthotropic
- reference configuration is needed

\[
\sigma = -pI + G(B - I) + T_{fo}(\lambda^2)e_fe_f
\]

\[
\lambda^2 = e_{fo} \cdot C \cdot e_{fo}
\]

\[
C = F^c \cdot F, \quad B = F \cdot F^c
\]
Mathematical model

power law neo-Hookean tissue:
(Truesdell, Raghavan, Di Martino)

- non-linear
- isotropic
- reference configuration is needed

\[
\sigma = -pI + 2(\alpha + 2\beta(\operatorname{tr} B - 3))(B - I)
\]
\[
B = F \cdot F^c
\]

\[
\alpha = O(0.2\text{MPa})
\]
\[
\beta = O(2\text{MPa})
\]
Mathematical model

/constitutive models

linear models:

- small strain modulus $E_s$
  - too large deformations
- linearized modulus $E_l$
  - too large deformations
- incremental modulus $E_i$
  - reasonable deformations
- reference configuration is needed

$$\sigma = -pI + G(B - I)$$
$$B = F \cdot F^c$$

$O(0.5\text{MPa}) < \alpha < O(5\text{MPa})$
Mathematical model

/circumferential stress:
\( p(a/h) = O(100 \text{ kPa}) \)

/residual stress:
\( E_s(h/a) = O(10 \text{ kPa}) \)

‘stress free’

imaging configuration
- stress state unknown
Mathematical model

stress [kPa]

initial stress
peak stress
residual stress

pressure

increment

imaging configuration
- stress state unknown

Mathematical model /nonlinear models
Mathematical model

/nonlinear models

stress [kPa]

initial stress

peak stress

residual stress

imaging configuration
- stress state unknown

pressure

increment

stretch ratio [-]

1.0 1.1 1.2 1.3 1.4

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Mathematical model /backward incremental method

stress [kPa]

pressure

increment

peak stress

initial stress

residual stress

imaging configuration - stress state unknown

F

stretch ratio [-]
Mathematical model

/backward incremental method

stress [kPa]

increment

presssure

peak stress

initial stress

residual stress

imaging configuration - stress state unknown

F

pressure

increment

stress [kPa]
**Mathematical model**

/backward incremental method

- stress [kPa]
- initial stress
- peak stress
- residual stress
- imaging configuration - stress state unknown
- pressure increment

- stretch ratio [-]
Mathematical model /backward incremental method

- stress [kPa]

- peak stress
- initial stress
- residual stress

Increment
- pressure

Imaging configuration
- stress state unknown

1.0 1.1 1.2 1.3 1.4

 Stretch ratio [-]
Mathematical model

/backward incremental method

- stress state unknown

- imaging configuration

- stress [kPa]

- pressure

- increment

- residual stress

- initial stress

- peak stress

$\mathbf{(F_1F_2)^{-1}}$

$1.0$ $1.1$ $1.2$ $1.3$ $1.4$

stretch ratio [-]
Mathematical model /backward incremental method

stress [kPa]

initial stress

peak stress

residual stress

imaging configuration - stress state unknown

pressure

increment

stretch ratio [-]

1.0 1.1 1.2 1.3 1.4

1.2

F
Mathematical model

/backward incremental method

stress [kPa]

pressure

increment

peak stress

initial stress

residual stress

\[(F_1 F_2 F_3)^{-1}\]

imaging configuration - stress state unknown

stretch ratio [-]

1.0 1.1 1.2 1.3 1.4
Mathematical model

/backward incremental method

stress [kPa]

pressure

increment

peak stress

initial stress

residual stress

imaging configuration - stress state unknown

F

1.0 1.1 1.2 1.3 1.4

stretch ratio [-]
Mathematical model /backward incremental method

stress [kPa]

pressure
increment

peak stress
initial stress
residual stress

$\left( F_1 F_2 F_3 F_4 \right)^{-1}$

imaging configuration - stress state unknown

stretch ratio [-]

$1.0 \, 1.1 \, 1.2 \, 1.3 \, 1.4$
Mathematical model
/backward incremental method

stress [kPa]

pressure

increment

peak stress

initial stress

residual stress

imaging configuration - stress state unknown

\( F_k \)

9.0 1.1 1.2 1.3 1.4 stretch ratio [-]
Mathematical model / backward incremental method

-stress [kPa]

-peak stress

-initial stress

-residual stress

-(F_1F_2F_3...F_k)^{-1}

-imaging configuration - stress state unknown

Mathematical model / backward incremental method

-stress [kPa]

-peak stress

-initial stress

-residual stress

-(F_1F_2F_3...F_k)^{-1}

-imaging configuration - stress state unknown
Mathematical model /backward incremental method

- stress [kPa]

- pressure increment

- residual stress

- peak stress

- initial stress

- imaging configuration - stress state unknown

- \( F_n \)

- stretch ratio [-]

- increment

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Mathematical model /backward incremental method

- stress state known

\[(F_1F_2F_3...F_kF_n)^{-1}\]

- imaging configuration
- stress state known

stress [kPa]

pressure

increment

initial stress

peak stress

residual stress

stretch ratio [-]

1.0 1.1 1.2 1.3 1.4
**morphology:**

- inhomogeneous structure
- properties that vary throughout the thrombus can be expected

(Wang et al., 2001)
Material properties

Type 1: Continuous transition
3 patients

Type 2: Discrete transition
4 patients

(Evelyne van Dam et al., Biorheology, 2007)
Material properties

morphological and mechanical characterization: wall

(Evelyne van Dam et al. 2005)
Material properties / arterial wall

**morphology:**
- A: adventitia 0.31
- M: media 0.24
- T: Thrombus 0.45

**modulus MPa:**
1 cluster:
- $E_{AMT} = 1.15$

2 clusters:
- $E_M = 1.49 \quad E_{AT} = 1.03$
- $E_{MT} = 0.16 \quad E_A = 3.40$
- $E_{MA} = 1.94 \quad E_T = 0.04$

3 clusters:
No convergence
Constitutive model:

\[ \sigma = -pI + 2(\alpha + 2\beta(\text{tr}B - 3))(B - I) \]

\[ B = F \cdot F^c \]

Methods:

1) Linear model: \( \beta=0, 2\alpha = G = 1 \text{ MPa} \quad (E_i= 3 \text{ MPa}) \)
2) Non-linear model: \( \beta=2, \alpha = 0.2 \text{ MPa} \quad (E_i= 3 \text{ MPa}) \)
3) Backward incremental method to include initial stress
Clinical study /method comparison

(Lambert Speelman et al. 2008)

99-percentile peak stress

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Material properties

Compliance measurements:

Subjects:
10 patients with AAA > 5.5 cm elective surgery/EVAR within 4 weeks

Method:
MRI slices and semi automatic segmentation
Pressure measurement: invasive, within AAA

Result:
Pressure-volume data

(Marcel van ‘t Veer et al., JVS, 2008)
Material properties

Pressure-Volume datapoints

PV-relation

best fit

\[ y = 0.048x + 203.8 \]

\[ R^2 = 0.96 \]
### Material properties

<table>
<thead>
<tr>
<th>Patient</th>
<th>C [ml/mmHg]</th>
<th>R²</th>
<th>Laplace:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.038</td>
<td>0.97</td>
<td>E=O(8 MPa) (≈ 3G)</td>
</tr>
<tr>
<td>2</td>
<td>0.085</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.046</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.027</td>
<td>0.83</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.050</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.037</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.038</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.048</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.031</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.0092</td>
<td>0.27</td>
<td></td>
</tr>
</tbody>
</table>
Clinical study / simulations vs patient data

Propagation and simulation contour

Simulation versus Propagation $\varepsilon_{\text{Mean}}$

- Blood Pressure [kPa]
- Heart Cycle [%]
- Prop Area [mm$^2$]

Propagation contour
Simulation contour

$\varepsilon_{\text{Mean}}$ [mm]

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Clinical study /stress vs diameter

High tortuosity

Low tortuosity

Linear Regression Model

<table>
<thead>
<tr>
<th>99 percentile of the stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
</tr>
<tr>
<td>330</td>
</tr>
<tr>
<td>300</td>
</tr>
<tr>
<td>270</td>
</tr>
<tr>
<td>240</td>
</tr>
<tr>
<td>210</td>
</tr>
</tbody>
</table>

Maximum AAA diameter (mm)

44 47 50 53 56 59
### Clinical study

<table>
<thead>
<tr>
<th>Stress</th>
<th>Tortuosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1.7 ± 0.7</td>
</tr>
<tr>
<td>Normal</td>
<td>1.2 ± 0.2</td>
</tr>
<tr>
<td>Low</td>
<td>1.2 ± 0.1</td>
</tr>
</tbody>
</table>

**/stress vs diameter**

**High stress AAAs**

**Low stress AAAs**
Clinical study /stress vs diameter

Follow up data: growth (each 4 month)
Clinical study /conclusions

Mathematical model
- Initial stress must be included to obtain realistic strain
- Backward incremental method can be used to obtain the initial state of stress
- Non-linear model must be used to obtain realistic systolic stress

Material properties
- AAA wall is strongly inhomogeneous
- Moduli of different parts of the wall are of the same order of magnitude
- Moduli found in this study correspond with literature data
- Thrombus is mechanically not very important

In-vivo compliance data
- Derived modulus is slightly higher than data obtained in-vitro

Clinical study
- Deformation obtained with non-linear model that includes initial stress corresponds well with MRI data
- 99-percent percentile peak stress correlates well with diameter
- Follow-up data may prove that stress analysis provides additional information about risk of rupture